Shadowing is a Structural Property

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Abstract

Let X be a compact metric space and $f : X \to X$ a continuous function. If you were to try to actually compute the orbit of a point, $x, f(x), f^2(x), f^3(x), \ldots$, you would end up with a pseudo-orbit, that is a sequence of points $x_1, x_2, x_3, \ldots$ such that the points $f(x_n)$ and x_{n+1} are close but not necessarily equal as the computation might not be exact. Such errors might accumulate, so that any attempt to compute the dynamics of the system are doomed to failure. However, this is not always the case; in certain systems pseudo-orbits are shadowed by real orbits with specifiable degree of accuracy. It turns out that shadowing has some interesting and well-documented consequences. We survey some of these results and discuss the close relationship between systems with shadowing and shifts of finite type, a class of system which turns up in many areas of mathematics.