Abstracts

Generalized Adler-Moser polynomials and multiple vortex rings for the Gross-Pitaevskii equation

Weiwei Ao
Wuhan University

We construct new finite energy traveling wave solutions with small speed for the three dimensional Gross-Pitaevskii equation. These solutions have the shape of 2n + 1 vortex rings, far away from each other. Among these vortex rings, n + 1 of them have positive orientation and the other n of them have negative orientation. The location of these rings are described by the roots of a sequence of polynomials with rational coefficients. The polynomials can be regarded as a generalization of the classical Adler-Moser polynomials. This is joint work with Yehui Huang, Yong Liu and Juncheng Wei.

Linear non-degeneracy and uniqueness of the bubble solution for the critical fractional Hénon equation in $\mathbb{R}^N$

Begoña Barrios
Universidad de La Laguna.
Tenerife. Canary Islands. Spain

In this talk we show a linear non-degeneracy result of positive radially symmetric solutions of the critical Henon equation associated to the fractional Laplacian operator in the N-dimensional Euclidean space; that is,

$$(-\Delta)^s u = |x|^\alpha u^{\frac{N+2s+2\alpha}{N-2s}} \text{ in } \mathbb{R}^N.$$

As a consequence a uniqueness result of those solutions with Morse index equal to one is obtained. In particular, we get that the ground state solution is unique. Our approach follows some ideas developed in the deep, and celebrated, papers done by R. Frank and E. Lenzmann (Acta Math. 2013) and R. Frank, E. Lenzmann, L. Silvestre (Comm. Pure Appl. Math. 2016), but, of course, our proofs are not based on ODE arguments as occurs in the local case. Our non-degeneracy result extends, in the radial setting, some known theorems proved by J. Dávila, M. del Pino and Y. Sire (Proc. Amer. Math. Soc. 2013) and by F. Gladiali, M. Grossi and S.L.N. Neves (Adv. Math. 2013). However, due to the nature of the fractional operator and the weight in nonlinearity, we also argue in a different way than these authors do.

The results presented in this talk have been obtained in collaboration with S. Alarcón and A. Quaas form U. Valparaíso (Chile).

A spinorial analogue of the Brezis-Nirenberg theorem

Thomas Bartsch
University of Giessen

Let (M, g, σ) be a compact Riemannian spin manifold of dimension $m \geq 2$, let $\mathbb{S}(M)$ denote the spinor bundle on M, and let D be the Atiyah-Singer Dirac operator acting on spinors $\psi : M \to \mathbb{S}(M)$. We present recent results on the existence of solutions of the nonlinear Dirac equation with critical exponent

$$D\psi = \lambda\psi + f(|\psi|)\psi + |\psi|^{\frac{2}{m-1}}\psi$$

where $\lambda \in \mathbb{R}$ and $f(|\psi|)\psi$ is a subcritical nonlinearity in the sense that $f(s) = o\big(s^{\frac{2}{m-1}}\big)$ as $s \to \infty$.

This is joint work with Tian Xu.

T. Bartsch, T. Xu: *A spinorial analogue of the Brezis-Nirenberg theorem.* J. Funct. Anal. **280** (2021), Article 108991

Prescribed curvature in the Lorentz-Minkowski space

Denis Bonheure
Research Francqui Professor - ULB

I will review the state of the art about the prescribed curvature equation in the Lorentz-Minkowski space. From the electrostatic Born-Infeld equation and the physical foundations to the up to date known results and open questions.

Minimizers of the $W^{s,1}$–fractional seminorm

Claudia Bucur
Università degli Studi dell'Insubria & Riemann International School of Mathematics

We will discuss some properties of minimizers of the $W^{s,1}$–fractional seminorm, that reflect those enjoyed by *functions of least gradient*, their classical counterparts. We investigate the connection between these minimizers and nonlocal minimal surfaces; we further study the asymptotic as $p \to 1$ of minimizers of the $W^{s,p}$-fractional seminorm and of its Euler-Lagrange equation, and the equivalence between minimizers of the $W^{s,1}$–energy and weak solutions of the fractional 1–Laplacian.
The results presented are obtained in collaboration with S. Dipierro, L. Lombardini, J. Mazón and E. Valdinoci.

Gradient estimates for nonlinear elliptic equations with a gradient-dependent nonlinearity

Florica Cîrstea
The University of Sydney

In this talk, we will discuss gradient estimates of the positive solutions to weighted p-Laplacian type equations with a gradient-dependent nonlinearity of the form

$$\mathrm{div}(|x|^{\sigma}|\nabla u|^{p-2}\nabla u) = |x|^{-\tau}u^q|\nabla u|^m \quad \text{in } \Omega^* := \Omega \setminus \{0\}. \tag{1}$$

Here, $\Omega \subseteq \mathbb{R}^N$ is a domain containing the origin, $N \geq 2$, $m, q \in [0,\infty)$, $1 < p \leq N+\sigma$ and $q > \max\{p-m-1, \sigma+\tau-1\}$. The main difficulty arises from the dependence of the right-hand side of (1) on x, u and $|\nabla u|$, without any upper bound restriction on the power m of $|\nabla u|$. Our proof of the gradient estimates is based on a two-step process relying on a modified version of the Bernstein's method. As a by-product, we extend the range of applicability of the Liouville-type results known for (1). The results are part of a joint paper with J. Ching (see [1]). We will also point out further applications of these gradient estimates to the classification of isolated singularities to related problems with a Hardy potential.

[1] J. Ching and F.-C. Cîrstea, Gradient estimates for nonlinear elliptic equations with a gradient-dependent nonlinearity. *Proc. Roy. Soc. Edinburgh Sect. A* **150** (2020), no. 3, 1361–1376.

Coupled and uncoupled sign-changing spikes of singularly perturbed elliptic systems

Mónica Clapp
Universidad Nacional Autónoma de México

We present some results on the existence and limit profile of solutions to a singularly perturbed system of elliptic equations having a prescribed number of positive and nonradial sign-changing components.
This is joint work with Mayra Soares (UNAM).

Billiards with refraction: an example in Celestial Mechanics

Irene De Blasi
University of Turin

A new type of dynamical system of physical interest, where two different forces act in two complementary regions of the plane separated by a regular closed curve, is considered: while in the inner region the dynamics is governed by the attraction of a Keplerian center, an harmonic oscillator acts in the outer one. The study of the orbits subjected to this composite potential is carried out following a broken geodesics technique, assuming that a refraction Snell's law, justified by variational considerations, acts on the interface to concatenate inner and outer arcs.

When the interface is radially symmetric (e.g. in the circular case), the system is completely integrable, and results in a rigid rotation on the circle. Its deformation, breaking the symmetry, leads to the rise of a more complex dynamics and possibly of chaotic behaviour; nevertheless, a particular class of period 1 orbits, the so-called homotetic ones, persist provided the interface satisfies suitable local properties.
The first part of the talk will analyse the local behaviour of the dynamics around such periodic trajectories, investigating their stability and their bifurcations in dependence of suitable parameters, and focusing in particular on the case of elliptic interfaces; as the inner Keplerian potential is singular at the origin, a suitable regularisation technique is needed.
The last part of the talk is devoted to the study of the global properties of the dynamics when the interface is a small perturbation of a circle; in this case, the orbits are studied in relation to their rotation number and KAM and Mather theories are taken into account.

The talk refers to the works in collaboration with Susanna Terracini.

On the number of critical points of solutions of elliptic problems

Fabio De Regibus
La Sapienza, Università di Roma

Let u be a classical solution of the problem

$$-\Delta u = f(u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

where f is smooth and $\Omega \subseteq \mathbb{R}^N$, $N \geq 2$, is a smooth and bounded domain. A classical problem concerns the study of the shape of u related to the one of the domain. In particular we investigate the number of critical points of u with respect to the convexity of the domain: if u is positive we discuss some generalization of preceding results involving the sign of the curvature of $\partial\Omega$, while if u is a sign-changing solution we show that the second eigenfunction of the Laplacian admits exactly two critical points if Ω is a planar and convex domain with large eccentricity.

Joint works with D. Mukherjee and M. Grossi.

The Bernstein technique for integrodifferential equations

Serena Dipierro
University of Western Australia

We present a version of the classical Bernstein technique for integro-differential operators. We provide first and one-sided second derivative estimates for solutions to fractional equations, including some convex fully nonlinear equations of order smaller than two, for which we prove uniform estimates as their order approaches two. Our method is robust enough to be applied to some Pucci-type extremal equations and to obstacle problems for fractional operators, although several of the results are new even in the linear case. The result discussed come from a joint work with Xavier Cabré and Enrico Valdinoci.

Sharp existence and classification results for nonlinear elliptic equations in $\mathbb{R}^N \setminus \{0\}$ with Hardy potential

Maria Farcaseanu
School of Mathematics and Statistics, The University of Sydney

We reveal the structure and asymptotic behavior near zero and infinity of all positive solutions for the nonlinear elliptic equation with Hardy potential $(\star)$ $-\Delta u - \frac{\lambda}{|x|^2} u + |x|^\theta u^q = 0$ in $\mathbb{R}^N \setminus \{0\}$ $(N \geq 3)$, where $q > 1$, $\theta \in \mathbb{R}$ and $\lambda \in \mathbb{R}$ are arbitrary. We provide the sharp range of the parameters such that there exist positive solutions of $(\star)$ in $\mathbb{R}^N \setminus \{0\}$. We show that equation $(\star)$ has either a unique solution or infinitely many solutions or no positive solutions. This is joint work with Florica Cîrstea.

Prescribed mass solutions to fractional Schrödinger equations

Marco Gallo
Università degli Studi di Bari Aldo Moro

In this talk we will discuss the problem of finding solutions to some fractional Schrödinger equation, focusing on the case where the mass of the particle is

prescribed in advance, but its frequency is unknown. The particular geometry of the Lagrangian product space, together with the general assumptions given on the nonlinearity and the lack of the regularizing effect of the fractional Laplacian, will lead to an approach where the use of the well known Pohozaev identity plays a key role. As a matter of fact, we will see how this identity models both the analysis and the geometry of the problem. The talk is based on a joint work with Silvia Cingolani and Kazunaga Tanaka.

On the uniqueness of positive solutions of the Lane-Emden problem in planar domains

Isabella Ianni
Università Sapienza Roma

The question of the uniqueness for the positive solutions of the Lane-Emden problem arose since the famous symmetry result by Gidas, Ni, Nirenberg (1979), which implies uniqueness when the domain is a ball. A conjecture on the uniqueness in any convex domain was then formulated during the eighties, but only partial answers have been given so far. In this talk we will describe recent results obtained in dim=2 about the asymptotic behavior of positive solutions, their non-degeneracy and Morse index computation. In particular we will show that the uniqueness conjecture in convex domains is true in the planar case and for p large enough.

This is from joint works with Francesca De Marchis, Massimo Grossi, Filomena Pacella (University Sapienza of Roma, Italy), Peng Luo and Shusen Yan (Wuhan Normal University, China).

Blaschke-Santalò diagram and eigenvalues of the Dirichlet-Laplacian

Jimmy Lamboley
Sorbonne université

The notion of Blaschke-Santalò diagram was mainly used in the context of geometry of planar convex sets, in order to describe all possible inequalities between geometric quantities like the area, the perimeter, the diameter, etc... We will start by describing this notion, and then see how one can apply it to understand the eigenvalues of the Laplace operator with Dirichlet boundary conditions. More precisely, we will describe the set

$$\Big\{(x,y)\in\mathbb{R}^2,\ \ \exists\,\Omega\in\mathcal{A},\ \ P(\Omega)=x,\ \lambda_1(\Omega)=y,\ |\Omega|=1\Big\}$$

where $|\cdot|$, $P(\cdot)$ and $\lambda_1(\cdot)$ are respectively the volume, the perimeter and the first Dirichlet eigenvalue of the laplacian of a set Ω. Also $\mathcal{A}$ denotes a class of sets, and we will mainly focus on two cases: the class of all open sets, and the class of planar convex sets.

This is a joint work with Ilias Ftouhi

Li and Yau estimates for some semilinear heat equations and applications

Carlo Mantegazza
Universitá di Napoli Federico II

We will show some Li and Yau–type gradient estimates for positive solutions of the semilinear heat equations $u_t = \Delta u + u^p$ with p¿1, on a complete n–dimensional Riemannian manifold (M,g) with nonnegative Ricci tensor. We then discuss some applications to ancient and eternal solutions. Joint work with Giacomo Ascione, Daniele Castorina and Giovanni Catino.

Overdetermined problems and shape optimization in cones

Filomena Pacella
Universitá di Roma Sapienza

We present some results about the problem of characterizing domains in cones for which a solution of a partially overdetermined problem exists. In recent papers in collaboration with G. Tralli it is proved that if the cone is convex the only domains with this property are spherical sectors centered at the vertex of the cone. This still holds if the cone is almost convex but it is not expected to be true for general nonconvex cones.
Some recent results in collaboration with A. Iacopetti and T. Weth show that this question is related to the study of the first nontrivial Neumann

eigenvalue of the Laplace-Beltrami operator on domains on the unit sphere allowing to determine classes of nonconvex cones for which the spherical sectors are not the minimizers for the associated shape optimization problem for the torsional energy functional.Then, using a concentration-compactness argument, we prove that minimizers for the shape optimization problem do exist in some cases, getting so nonradial solutions of the overdetermined problem.

On the smooth convergence of geometric flows

Marco Pozzetta
Università di Napoli Federico II

An extrinsic geometric energy is a functional $\mathcal{E}$ defined on smooth immersions $\gamma : M \hookrightarrow \mathbb{R}^n$ of a given manifold M depending on extrinsic geometric quantities. An extrinsic geometric flow is a smooth family of immersions $\gamma_t : M \hookrightarrow \mathbb{R}^n$, for $t \in [0, T)$, such that a given extrinsic geometric energy $\mathcal{E}$ evaluated on γ_t decreases in time by some assigned evolution equation. If such a geometric flow γ_t is defined for any positive time, i.e., $T = +\infty$, we are interested in the existence of a limit smooth immersion $\gamma_\infty = \lim_{t\to+\infty} \gamma_t$. In this talk we present a strategy for promoting the sub-convergence of such a flow, that is the existence of a limit $\lim_k \gamma_{t_k}$ up to isometries of $\mathbb{R}^n$ and on suitable diverging sequences t_k, into the existence of the full limit γ_∞. We analyze the explicit example of the L^2-gradient flow of the elastic energy $\mathcal{E}(\gamma) := \int_{\mathbb{S}^1} 1 + \frac{1}{2}|k|^2 \,\mathrm{d}s$, where $\gamma : \mathbb{S}^1 \hookrightarrow \mathbb{R}^n$ is an immersion of the circle $\mathbb{S}^1$ and k is the curvature vector of γ. The strategy of the proof is based on the application of a Łojasiewicz–Simon gradient inequality. We discuss how the same argument can be applied also for deducing the smooth convergence of the gradient flow of the p-elastic energy of closed curves in arbitrary Riemannian manifolds for $p \in [2, +\infty)$, and of the gradient flow of suitable higher order geometric flows of closed hypersurfaces.

Part of the results are obtained in collaboration with Carlo Mantegazza.

Blowing-up solutions for second-order critical elliptic equations: the impact of the scalar curvature

Frédéric Robert
Université de Lorraine (France)

Given a closed manifold (M^n, g), $n \geq 3$, Olivier Druet proved that a necessary condition for the existence of energy-bounded blowing-up solutions to perturbations of the equation

$$\Delta_g u + h_0 u = u^{\frac{n+2}{n-2}},\ u > 0 \text{ in } M$$

is that $h_0 \in C^1(M)$ touches the Scalar curvature somewhere when $n \geq 4$ (the condition is different for $n = 6$). In this paper, we prove that Druet's condition is also sufficient provided we add its natural differentiable version. For $n \geq 6$, our arguments are local. For the low dimensions $n \in \{4, 5\}$, our proof requires the introduction of a suitable mass that is defined only where Druet's condition holds. This mass carries global information both on h_0 and (M, g).

Joint work with Jérôme Vétois (McGill University, Canada)

The Gaussian and geodesic prescription problem via conformal changes of the disk.

David Ruiz
IMAG, University of Granada

The problem of prescribing the Gaussian curvature $K(x)$ on compact surfaces is a classic one, and dates back to the works of Berger, Moser, Kazdan and Warner, etc. The case of the sphere receives the name of Nirenberg problem and has deserved a lot of attention in the literature. In the first part of the talk we will review the known results about compactness and existence of solutions for that problem.

If the domain has a boundary, the most natural question is to prescribe also the geodesic curvature $h(x)$ of the boundary. This problem reduces to solve a semilinear elliptic PDE under a nonlinear Neumann boundary condition. In this talk we focus in the case of the standard disk.

First we perform a blow-up analysis for the solutions of this equation. We will show that, if a sequence of solutions blow-up, it tends to concentrate around a unique point in the boundary of the disk. We study also the location of such point, which concerns both curvature terms due to the interaction between them. Quite interestingly, such conditions depend on $h(x)$ in a nonlocal way. This is joint work with A. Jevnikar, R. López-Soriano and M. Medina.

Secondly, we will give existence results. We will show how the blow-up analysis developed before can be used to compute the Leray-Schauder degree associated to the problem in a compact setting.

Heat content, exit time moments and their critical domains

Alessandro Savo
Università di Roma, La Sapienza

We discuss the heat content functional of a domain with smooth boundary in a general Riemannian manifold. In particular, we characterize its critical domains by the existence of an isoparametric foliation, that is, a foliation by parallel, CMC hypersurfaces collapsing onto a unique singular minimal leaf (the focal submanifold). This leads to a complete classification when the ambient manifold is a space form, in particular, a round sphere. We then plan to discuss the relation of heat content with exit time moments, thus obtaining a geometric classification of critical domains as well.

Mapping solutions to the constraint equations

Caterina Valcu
École Polytechnique de Paris, Francia

We study initial data in General Relativity, which are defined as solutions to the constraint equations. The focus in this talk is a modified version of the conformal method proposed by David Maxwell. While the model seems more strongly justified from a geometrical standpoint, the resulting system becomes significantly more difficult to solve; it presents critical nonlinear terms, including gradient terms. We describe existence and stability while working in dimensions 3,4 and 5, under smallness assumptions and in the presence of a scalar field with positive potential. The tools we use are related to obtaining a priori estimates (compactness results) and a fixed-point theorem.

Long-time behavior for the porous medium equation with small initial energy

Bruno Volzone
Dipartimento di Scienze e Tecnologie, Università degli Studi di Napoli "Parthenope"

In this talk,we will describe some aspects of a study developed in a joint paper with L. Brasco concerning the long-time behavior for the solution of the Porous Medium Equation in an open bounded connected set, with smooth boundary and sign-changing initial datum. Homogeneous Dirichlet boundary conditions are considered. We prove that if the initial datum has sufficiently small energy, then the solution converges to a nontrivial constant sign solution of a sublinear Lane-Emden equation, once suitably rescaled.

We also give a sufficient energetic criterion on the initial datum, which permits to decide whether convergence takes place towards the positive solution or to the negative one.