Workhop "Geometric Analysis and PDEs in Sorrento II"

Abstract

Recent developments in quantitative rearrangement inequalities

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In this talk we address the challenge of providing quantitative versions of inequalities related to symmetrization of functions. In particular, we will focus on new quantitative versions of the Talenti inequality. We will study the issue of finding sharp versions of this stability inequality, giving some positive result in some special cases.

Some recent results for singular anisotropic elliptic equations

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In this talk, we discuss some recent results on the existence and uniform boundedness of solutions for a general class of Dirichlet anisotropic elliptic problems of the form

$$-\Delta_{\overrightarrow{p}} u + \Phi_0(u, \nabla u) = \Psi(u, \nabla u) + f \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

where Ω is a bounded open subset of $\mathbb{R}^N$ ($N \geq 2$), $\Delta_{\overrightarrow{p}} u = \sum_{j=1}^N \partial_j(|\partial_j u|^{p_j-2}\partial_j u)$ and $\Phi_0(u, \nabla u) = \left(\mathfrak{a}_0 + \sum_{j=1}^N \mathfrak{a}_j |\partial_j u|^{p_j}\right) |u|^{m-2}u$, with $\mathfrak{a}_0 > 0$, $m, p_j > 1$, $\mathfrak{a}_j \geq 0$ for $1 \leq j \leq N$ and $N/p = \sum_{k=1}^N (1/p_k) > 1$. We assume that $f \in L^r(\Omega)$ with $r > N/p$. The feature of this study is the inclusion of a possibly singular gradient-dependent term $\Psi(u, \nabla u) = \sum_{j=1}^N |u|^{\theta_j-2}u\,|\partial_j u|^{q_j}$, where $\theta_j > 0$ and $0 \leq q_j < p_j$ for $1 \leq j \leq N$.

Based on joint works with Florica C. Cîrstea (The University of Sydney, Australia).

Sharp Poincaré Inequalities in possibly non-convex domains

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We establish sharp estimates for the first nonzero eigenvalue of the Neumann p-Laplacian in possibly non-convex domains. Two distinct approaches are employed: one based on symmetrization methods, and the other leveraging a well-known technique introduced by Payne and Weinberger for convex sets. (Joint papers with B. Brandolini, C. Trombetti, E. Dryden and J. Langford)

On a double-symmetrization inequality for anisotropic integrals of Sobolev functions

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This talk concerns a Pólya-Szegö type inequality for anisotropic functionals of Sobolev functions. The relevant inequality entails a double-symmetrization involving both trial functions and functionals. A new approach that uncovers geometric aspects of the inequality is proposed. It relies upon anisotropic isoperimetric inequalities, fine properties of Sobolev functions, and results from the Brunn-Minkowski theory of convex bodies. Importantly, unlike previously available proofs, ours does not require approximation arguments and hence allows for a characterization of extremal functions. This is a joint work with Gabriele Bianchi and Paolo Salani.

A spectral isoperimetric inequality on the n-sphere for the Robin-Laplacian with negative boundary parameter

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For every given $\beta < 0$, we study the problem of maximizing the first Robin eigenvalue of the Laplacian $\lambda_\beta(\Omega)$ among convex (not necessarily smooth) sets $\Omega \subset \mathbb{S}^n$ with fixed perimeter. In particular, we show that geodesic balls maximize the eigenvalue for any fixed perimeter P smaller than the one of an n-dimensional hemisphere.

Optimization problems in models for heat transfer

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In this talk, I will present some shape optimization problems for elliptic equations with Robin boundary conditions. Problems of this type, in the case of Dirichlet conditions, have been widely studied in the literature. In this presentation, I will focus on the case of Robin boundary conditions. Interest in these types of problems arises from their application in heat transfer models related to thermal insulation. After illustrating some of these models, I will present the results of recent research on shape optimization in this context. The results I will describe are part of works in collaboration with Carlo Nitsch, Francescantonio Oliva, Riccardo Scala, and Cristina Trombetti.

Asymptotic behaviour for some anisotropic equations

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We consider the evolution Finsler p-Laplacian in $\mathbb{R}^N$ in the range $p > \frac{2N}{N+1}$. We analyze the self-similarity and we give some details on the existence and uniqueness of the nonnegative solutions for the Cauchy problem with an initial nonnegative L^1 datum. The asymptotic behaviour of such solutions is also described under some suitable assumptions. Moreover we use some symmetrization results in order to derive an $L^1 - L^\infty$ estimate.

A measure for the stability of structures immersed in a 2D laminar flow

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We introduce a new measure for the stability of structures, such as the cross-section of the deck of a suspension bridge, subject to a 2D fluid force, such as the lift exerted by a laminar wind. We consider a wide class of possible flows, as well as a wide class of structural shapes. Within a suitable topological framework, we prove the existence of an optimal shape maximizing the stability. Applications to engineering problems are also discussed. Based on joint works with Edoardo Bocchi, Politecnico di Milano.

Around the 1/2 conjecture for the Neumann-Poincaré operator

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We study the variation of the eigenvalues of the Neumann-Poincaré operator when the support of integration is perturbed. The focus is on the so-called 1/2 conjecture, which is linked to minimizing the second eigenvalue in three dimensions, and its natural generalization to higher dimensions.

Based on a joint work with Matteo Dalla Riva, Paolo Luzzini and Paolo Musolino.

On a singular nonlinear elliptic problem in the half-space

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Under minimal assumption, we provide a classification result for positive solutions to $-\Delta u = 1/u^\gamma$ in the half space, with zero Dirichlet boundary condition.

References

[1] L. Montoro, L. Muglia, B. Sciunzi, Classification of solutions to $-\Delta u = u^{-\gamma}$ in the half-space, Math. Ann. 389 (2024), no. 3, 3163-3179.

[2] L. Montoro, L. Muglia, B. Sciunzi, The Classification of all weak solutions to $-\Delta u = u^{-\gamma}$ in the half-space, arXiv:2404.03343v1

Widely degenerate parabolic problems

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We present some regularity results for the gradient of the weak solutions to strongly degenerate parabolic PDE's that arise in gas filtration problems . More precisely, we will present a continuity result of a suitable function of the gradient as well as a second order regularity result of a nonlinear function of the spatial gradient Du, which in turn implies the existence of the weak time derivative. The results are obtained in collaboration with V. Bogelein (Salzburg University), F. Duzaar (Salzburg University) and R. Giova (University of Naples Parthenope) and with P. Ambrosio (University of Naples Federico II) and A. Gentile (Università Politecnica delle Marche).

On the gradient regularity of the solutions to the p-Laplacian parabolic equation

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In this talk we describe the in uence of the initial data and the forcing terms on the regularity of the solutions to a class of evolution equations including the heat equation, linear and semilinear parabolic equations, together with the nonlinear p-Laplacian equation. We focus our study mainly on the regularity (in terms of belonging to appropriate Lebesgue spaces) of the gradient of the solutions.

Optimization problems arising in populations dynamic

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We will discuss some recent results concerning optmization problems arising in populations dynamic models. The optimization of the distribution of resources in logistc models leads to minimize a principal eigenvalue with respect to the sign-changing weight. Important qualitative properties of the positivity set of the optimal weight, such as being connected, as well as its location, are still not known in general. We will present some new achievements in the asymptotical study regarding these properties. When the model predicts survival for every diffusion coefficient, it becomes relevant the optimization of the total population, which is more related to the nonlinear problem. Joint works with Francesca Gladiali (UniversitÃ di Sassari), Iula Martina Bulai (UniversitÃ di Sassari), Dario Mazzoleni (UniversitÃ di Pavia), Lorenzo Ferreri (Scuola Normale Superiore di Pisa), Gianmaria Verzini (Politecnico di Milano).

Concavity principles for the principal frequency in Gauss Space

Paolo Salani

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I will prove a Brunn-Minkowski inequality for the Dirichlet eigenvalue of the Ornstein-Uhlenbeck operator and the log-concavity of the associated eigenfunction of a convex domain.
The talk is based on a joint work with A. Colesanti, E. Francini, G. Livshyts.

Classification of solutions to Hardy-Sobolev doubly critical systems

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I will present a result about the classification of the solutions to a family of Hardy-Sobolev doubly critical system defined in the whole space. More precisely, we provide the classification of the positive solutions, whose expressions comprise multiplies of solutions of the decoupled scalar equation. This is a work in collaboration with Francesco Esposito and Rafael Lòpez-Soriano.

On overview on nonlinear Schrödinger systems

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In this talk I will present some recent existence results on nonlinear Schrödinger type systems in a weak fully attractive or repulsive regime in presence of an external trapping potential with subcritical ora critical growth.

On some noncoercive nonlinear problems

Gabriella Zecca

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In this talk I will focus on some nonlinear and non-coercive second-order differential operators, whose model appears in reaction-diffusion problems. I will present existence and regularity results for the solutions of the parabolic Dirichlet problems for such operators, under 'optimal' assumptions on the *convection* term. The obstacle problem for such operators is also considered, when the obstacle function is irregular with respect to the time variable. These results have been obtained in collaboration with F. Farroni, L. Greco and G. Moscariello.